

Propositional Logic – Exercises for Week 2

1. Can a valid argument have:

- i. true premisses and a false conclusion?
- ii. false premisses and a true conclusion?
- iii. false premisses and a false conclusion?
- iv. some true premisses, some false premisses, and a true conclusion?
- v. some true premisses, some false premisses, and a false conclusion?

In each case either give an example or explain why not.

2. What can you say about the validity of an argument that has:

- i. premisses consistent with the conclusion?
- ii. premisses inconsistent with the negation of the conclusion?
- iii. premisses inconsistent with one another?
- iv. a tautologous (necessarily true) conclusion?
- v. an inconsistent (necessarily false) conclusion?

3. For each of the following arguments, provide the *counterexample set* – the set consisting of the premisses plus the negation of the conclusion – and say whether this set is consistent. If the counterexample set is *consistent*, outline a situation in which all of its members are true. If it is *inconsistent*, explain why. What does this tell you about the validity of each argument?

- i. Either Alice or Brian is guilty.
If Brian is guilty, then Alice is guilty too.
Therefore Alice is guilty.
- ii. Either Cathy is lying, or Diana is at home.
If Ellen is right, then Cathy is lying.
Therefore either Ellen is not right, or Diana is at home.
- iii. If Fred didn't play, then George didn't play either.
George did play.
Therefore both Fred and George played.
- iv. If Helen went, so did Isabel.
If Isabel went, so did Jane.
If Jane went, Isabel didn't go.
Therefore neither Isabel nor Helen went.
- v. If Kevin has a car, so does Leo.
If Leo has a car, so does Mike.
If Mike has a car, Kevin doesn't.
Therefore neither Leo nor Kevin has a car.
- vi. Either Nancy is rich, or Oswald is deceived.
If Oswald is deceived, then Nancy is lying.
Nancy isn't lying, and she isn't rich.
Therefore Oswald should ditch Nancy for Pauline.

- vii. Quentin studies philosophy.
Rachel studies classics.
Therefore either Susan studies physics or she doesn't study physics.
- viii. All cows like aubergines.
Daisy is a cow.
Therefore Daisy likes aubergines.
- ix. All intelligent martians are green.
Therefore some intelligent martians are green.

4. Write out truth-tables for these formulae:

- i. $[P \rightarrow Q]$
- ii. $[\neg P \rightarrow Q]$
- iii. $[P \rightarrow \neg Q]$
- iv. $[P \rightarrow P]$
- v. $[P \rightarrow \neg P]$
- vi. $\neg[P \rightarrow Q]$
- vii. $\neg[Q \rightarrow \neg P]$
- viii. $[P \wedge \neg[Q \rightarrow R]]$
- ix. $[[P \wedge \neg Q] \rightarrow R]$
- x. $[[P \rightarrow Q] \vee [\neg P \rightarrow Q]]$

Are any of the above, or of last week's formulae (q. 2), tautologies? Are any inconsistent?

5. What is scope ambiguity? Provide an example, and clearly disambiguate it.