

Propositional Logic – Exercises for Week 3

1. For each of the arguments from last week's q. 3, (a) translate the premisses and conclusion into propositional notation, giving an appropriate key, and (b) use a truth-table to establish whether the argument is valid, clearly stating your result. Here is (iii) as a worked example:

Key	F:	Fred played			
	G:	George played			
Translation		$[\neg F \rightarrow \neg G]$			
		G			
	\therefore	$[F \wedge G]$			
Truth-Table	F	G	$[\neg F \rightarrow \neg G]$	G	$[F \wedge G]$
	T	T	F	T	T
	T	F	F	T	F
	F	T	T	F	F
	F	F	T	T	F

There is no line of the truth-table in which the premisses are true and the conclusion is false; the argument is therefore *valid*.

2. Give an example of an argument with a conclusion irrelevant to the premisses, and:
- i. a tautological conclusion.
 - ii. an inconsistent set of premisses.

Are the arguments you have just given valid or invalid, given the standard definitions?

3. Draw up truth-tables – partial or complete, as appropriate – for each of the following sentence-functors, and specify which of them, if any, can be interpreted as truth-functors:

- | | |
|--------------------------------------|-----------------------------------|
| i. ϕ and ψ . | v. ϕ if and only if ψ . |
| ii. ϕ but nevertheless ψ . | vi. ϕ unless ψ . |
| iii. If ϕ then ψ . | vii. ϕ because ψ . |
| iv. ϕ only if ψ . | viii. If ϕ then ϕ . |

Justify any question-marks in your truth-tables by giving two example substitutions with the same 'input' truth-value which nevertheless result in different 'output' truth-values for the compound proposition. Is anything lost in interpreting any of the above as truth-functors?

4. Write out truth-tables for these formulae:

- i. $[P \leftrightarrow Q]$
- ii. $[P \vee [\neg Q \leftrightarrow R]]$
- iii. $[[P \rightarrow R] \rightarrow [[Q \rightarrow R] \rightarrow [[P \vee Q] \rightarrow R]]]$

5. How many rows are there in an n-place truth-table? (E.g. a 2-place truth-table has 4 rows: TT-, TF-, FT-, FF-.) How many different n-place truth-tables does this mean there are?