

Propositional Logic – Exercises for Week 4

1. Produce tableaux for the following formulae, and list the structures in which they are true:

- i. $[P \wedge \neg P]$
- ii. $[\neg P \vee [Q \rightarrow P]]$
- iii. $[\neg P \rightarrow [P \rightarrow Q]]$
- iv. $[[P \vee \neg Q] \wedge \neg[\neg P \rightarrow \neg Q]]$
- v. $[\neg[P \wedge Q] \rightarrow [\neg P \wedge \neg Q]]$

(Structures should be *fully* specified as sets of true propositions, e.g. $\{P, \neg Q\}$ or $\{\neg P, \neg Q\}$.)

2. Give two symbolizations of each of these, only one of which reveals the inconsistency:

- i. I managed to produce tomatoes, and I didn't produce any tomatoes.
- ii. Sydney isn't the capital of Australia, but we all know that it is.
- iii. Bill and Ted went to the party; Ted was at the party but Bill wasn't.
- iv. Achilles is going to win the race, but he won't.
- v. Richard is a married bachelor.

3. Formalize the following arguments as *syntactic sequents* of the propositional calculus, giving an explicit interpretation of your sentence-letters and using the syntactic turnstile \vdash :

- i. The murderer was either Colonel Mustard or Professor Plum. But it wasn't Professor Plum. So it was Colonel Mustard.
- ii. Either the Master or the Dean was in the library. But if the Master wasn't there, the Dean wasn't there either. So they were both in the library.
- iii. You can only buy a Young Persons railcard if you're under 26 or a student; otherwise not. If you can buy a Young Persons railcard, you can get discounted train tickets. But you're not under 26. So unless you're a student, you can't get discounted train tickets.
- iv. If God is willing to prevent suffering, but unable to do so, He is not omnipotent. If He is able to prevent suffering, but unwilling to do so, He is not loving. If God exists, He is loving and omnipotent. And if He is both willing and able to prevent suffering, then there can't be any suffering – but there is. So God doesn't exist.
- v. The protesters will go away if Oxford stops experiments on animals. But this could only happen with government intervention. So, unless the government intervenes, they won't go away.

4. Produce tableaux for the sequents you have just given. What does each tableau tell you about the correctness of the relevant sequent, and about the validity of the original argument?

5. Draw up truth-tables for these sentence-functors, and say whether they are truth-functors:

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| i. Although ϕ, ψ . | v. I believe that ϕ . |
| ii. The fact that ϕ suggests that ψ . | vi. I know that ϕ . |
| iii. It is logically possible that ϕ . | vii. Omniscient Zeus believes that ϕ . |
| iv. It is logically necessary that ϕ . | viii. ϕ . |