

Propositional and Predicate Logic – Exercises for Week 5

1. Produce tableaux for the following formulae, and list the structures in which they are true:

- i. $[\neg[P \rightarrow Q] \rightarrow [P \wedge \neg Q]]$
- ii. $[[P \vee \neg[Q \wedge R]] \vee [[Q \rightarrow R] \rightarrow P]]$
- iii. $[[[P \wedge Q] \rightarrow R] \rightarrow [[P \rightarrow R] \wedge [Q \rightarrow R]]]$
- iv. $[[[P \vee Q] \rightarrow R] \rightarrow [[P \rightarrow R] \wedge [Q \rightarrow R]]]$
- v. $[[[P \rightarrow Q] \wedge [R \rightarrow S]] \rightarrow [[P \vee R] \rightarrow [Q \vee S]]]$

2. Use tableaux to show that the following syntactic sequents are correct:

- i. $[P \wedge \neg P] \vdash Q$
- ii. $P \vdash [Q \vee \neg Q]$
- iii. $[[P \vee Q] \wedge \neg Q] \vdash P$
- iv. $\vdash [[[P \wedge \neg Q] \rightarrow \neg P] \leftrightarrow [\neg Q \rightarrow [\neg P \wedge \neg Q]]]$
- v. $\vdash [[[P \wedge Q] \rightarrow R] \leftrightarrow [[P \rightarrow R] \vee [Q \rightarrow R]]]$

3. Provide counterexamples to show that the following semantic sequents are incorrect:

- i. $[P \rightarrow Q] \vDash [\neg P \rightarrow \neg Q]$
- ii. $[P \wedge [Q \vee R]] \vDash [[P \wedge Q] \vee R]$
- iii. $[[P \rightarrow Q] \wedge [P \rightarrow \neg Q]] \vDash$
- iv. $\vDash [[[\neg P \vee R] \wedge [R \vee \neg Q]] \wedge [Q \vee \neg P]]$
- v. $\vDash [[[P \wedge Q] \rightarrow R] \leftrightarrow [[P \rightarrow R] \wedge [Q \rightarrow R]]]$

4. Formalize the following as a syntactic sequent and assess its validity with a tableau:

If the hog-sprocket is defective, the bendix is not going to function. And if the grunge pedal is defective, the thermoblaster is not going to function. If the cramp-iron is not working, it must be because the hog-sprocket is defective. And if the colour detector is not working, it must be because the grunge pedal is defective. The cramp-iron and the colour detector are not both working. So either the bendix or the thermoblaster is not going to function.

5. Formalize the following using predicate formulae, giving domains and interpretations:

- i. *The ‘cows’ argument from Week 2, question 3 (viii).*
- ii. *The ‘martians’ argument from Week 2, question 3 (ix).*
- iii. *There is a day before all days. (This is ambiguous, so give two formalizations.)*
- iv. *Every good boy deserves favour.*
- v. *No one likes everyone.*
- vi. *If everyone is special, no one is special.*
- vii. *Anyone who doesn’t like garlic is a vampire.*
- viii. *The only good logician is a dead logician.*