

Logic – Exercises for, er, Week 9

1. Provide counterexamples to the following incorrect semantic sequents (see worked example):

- i. $\exists x[Fx \wedge Gx] \vDash$
- ii. $\forall x[Fx \vee Gx] \vDash$
- iii. $\exists x[Fx \rightarrow \neg Fx] \vDash$
- iv. $\forall x[Fx \leftrightarrow \neg Fx] \vDash$
- v. $\vDash \forall xFx$
- vi. $\vDash \forall x[\neg Fx \rightarrow \neg Gx]$
- vii. $Fa, \forall x[Fx \rightarrow Gx] \vDash$
- viii. $Fa, \exists x[Gx \wedge \neg Fx] \vDash$
- ix. $Fa, \forall xGx \vDash$
- x. $Fa, \forall x[\neg Gx \rightarrow \neg Fx] \vDash$
- xi. $\forall xFx, \exists x\exists y \neg x=y \vDash$
- xii. $\forall xFx \vDash \exists xFx$
- xiii. $\exists xFx \vDash \forall xFx$
- xiv. $\forall x[Fx \vee Gx] \vDash \forall x[\neg Fx \vee \neg Gx]$
- xv. $Fa, \forall x[Fx \rightarrow Gx] \vDash \forall xGx$
- xvi. $\forall xRxx, \forall x\forall y[Rxy \rightarrow Ryx] \vDash \forall x\forall y\forall z[[Rxy \wedge Ryz] \rightarrow Rxz]$
- xvii. $[\forall xFx \leftrightarrow \forall xGx] \vDash \forall x[Fx \leftrightarrow Gx]$
- xviii. $\forall x[[Fx \wedge Gx] \rightarrow Rax] \vDash \exists x[Rax \vee [Fx \vee Gx]]$
- xix. $\forall x[Fx \rightarrow \forall y[Gy \rightarrow Rxy]], \forall x[Hx \vee \neg\exists y[Fy \wedge Rxy]], \forall x\forall y[Rxy \rightarrow Ryx] \vDash \exists x[Fx \rightarrow \forall y[Gy \rightarrow Hy]]$
- xx. $\exists x x=x, \forall x\exists y[Fx \rightarrow [Fy \wedge \neg x=y]] \vDash \exists x\exists y[[Fx \wedge Fy] \wedge \neg x=y]$

Worked example: $Fa, \exists x[Fx \rightarrow \forall yGy] \vDash Ga$

For a counterexample, we need Fa and $\exists x[Fx \rightarrow \forall yGy]$ to be true, and Ga to be false. We can't have $[Fa \rightarrow \forall yGy]$ true, because (since Fa is true) that would make $\forall yGy$ and therefore Ga true. So we need another object, b , with Fb false and thus $[Fb \rightarrow \forall yGy]$ true. Our counterexample is then:

Domain = $\{a, b\}$

$F = \{a\}$

$G = \emptyset$

2. Provide counterexamples to the following incorrect semantic sequents:

- i. $[P \rightarrow \neg[Q \rightarrow R]], [R \vee Q] \vDash [P \leftrightarrow [Q \wedge \neg R]]$
- ii. $[[P \rightarrow Q] \wedge [R \rightarrow S]] \vDash [[P \wedge Q] \rightarrow [R \wedge S]]$
- iii. $[P \wedge [Q \rightarrow R]] \vDash [S \wedge [P \leftrightarrow Q]]$
- iv. $\vDash [[P \rightarrow Q] \rightarrow R] \leftrightarrow [[\neg P \wedge \neg Q] \vee \neg[Q \wedge \neg R]]$
- v. $\vDash [[P \vee Q] \rightarrow R] \leftrightarrow [[P \rightarrow R] \vee [Q \rightarrow R]]$